

# **WP5: Optimal Dispatch of Integrated Energy System**

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**Optimal Dispatch of Integrated Energy System**

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# Content

1.	Introduction .....	4
2.	Mathematical formulation.....	5
2.1	Notation.....	5
2.2	Linkages among power, gas, and heat subsystems.....	8
2.3	Operation constraints of the EPS.....	9
2.4	Operation constraints of the NGS.....	10
2.5	Optimal operation of the DHS.....	11
3.	Case study .....	12
3.1	Network operational parameters .....	13
3.2	Scheduling of the energy sources .....	14
4.	Summary .....	16
5.	Reference.....	17

# 1. Introduction

In recent years, research investigations have demonstrated that the integration of energy systems can balance the energy production and consumption in a broader scope, and hence improve the efficiency and sustainability of the energy utilization [1]. Therefore, the investigations of multi-energy systems (MES) are currently receiving increasing attention.

Since electricity and natural gas are two of the common options for bulk energy transmission, extensive studies have been carried out to investigate the coordinated operation of the gas and power system. [2] develops a steady-state model for the integrated gas and power systems, while [3] develops a dynamic energy flow model which considers the different response times of the gas and power systems. [4] proposes a coordinated scheduling strategy to optimize conflicting benefits of the electricity and gas networks. [5] proposes a bi-level dispatch model to minimize the total operating costs of both natural gas and electricity systems.

Several works have also been conducted in the coordination of the electrical system and heating system, due to the extensive use of CHP units, heat pumps, and electric boilers. [6] proposes an optimization model to coordinate the electrical and heating systems to accommodate the renewable sources. [7] proposes a combined heat and power dispatch model to operate the electric power system and district heating system. [8] develops a transmission-constrained unit commitment model on the combined electricity and district heating networks. These studies suggest that the coordinated operation can enhance the flexibility of the power system and accommodate high penetration level of renewable energy generation.

Although the coordination of the gas and power systems and the coordination of electrical and heating system have been studied well, there is few work on the joint operation of electricity, gas, and district heating systems. [9] presents a steady state power flow model for combined optimization of electricity, gas, and district heating systems based on the concept of energy hubs. However, it ignores the detailed network constraints of the electricity, gas and district heating system.

This WP is to develop the mathematical model of the integrated energy system which includes the electrical power system (EPS), the district heating system (DHS) and the natural gas system (NGS). The major contribution is providing an optimization problem for the joint operation of electricity, gas, and district heating systems. The objective is to minimize the operational costs of the integrated systems while maximizes the renewable energy consumed.

The remainder of the report is organized as follows. Section 2 presents a model to describe the optimization problem for joint operation of the electricity, gas and heating systems. Section 3 analyzes a case study. Finally, Section 4 gives conclusions.

## 2. Mathematical formulation

### 2.1 Notation

#### *Sets and Indices*

$\Omega_n^{\text{ED}}$	Set of electricity demands at bus $n$
$\Omega_n^{\text{GC}}$	Set of gas compressors at node $n$
$\Omega_n^{\text{GD}}$	Set of gas demands at node $n$
$\Omega_n^{\text{ST}}$	Set of gas storages at node $n$
$\Omega_n^{\text{GS}}$	Set of gas sources at node $n$
$\Omega_n^{\text{HD}}$	Set of heat demands at node $n$
$\Omega_n^{\text{HP}}$	Set of heat pumps at node $n$
$\Omega_n^{\text{HS}}$	Set of heat storages at node $n$
$\Omega_n^{\text{LP}}$	Set of linepack pipeline at node $n$
$\Omega_n^{\text{P2G}}$	Set of P2G unit at node $n$
$\Omega_n^{\text{WF}}$	Set of wind farms at bus $n$
$\Lambda^{\text{EPS}}$	Set of buses in the electric power subsystem (EPS)
$\Lambda^{\text{NGS}}$	Set of nodes in the natural gas subsystem (NGS)
$\Lambda^{\text{DHS}}$	Set of nodes in the district heating subsystem (DHS)
$\Lambda_n$	Set of buses directly connected to bus $n$ by transmission lines
$T$	Set of hours in the study horizon
$x$	Set of decision variables
REF	Reference bus with phase angle fixed to 0

#### *Constants*

$A$	the incidence matrix of the network
$B_{nm}$	Susceptance of transmission line $n-m$ [p.u.]
$c$	The specific heat capacity of water
$COP$	The coefficient of performance of the heat pump
$C_i^{\text{CFP}}$	Marginal cost of power production from CFP unit $i$ [\$/MWh]

$C_h^{HS,in/out}$	Marginal cost of heat input/output to/from heat storage $s$ [\$/MWh]
$C_s^{ST,in/out}$	Marginal cost of gas input/output to/from gas storage $s$ [\$/MWh]
$C_g^{GS}$	Marginal cost of the natural gas supplied from gas source $g$ [\$/MWh]
$d_{mn} \cdot l_{mn}$	The diameter and length of pipe $m-n$ [m]
$D_{d,t}^{ED}$	Hour- $t$ electricity demand at bus $d$ [MW]
$D_{d,t}^{GD}$	Hour- $t$ gas demand at gas node $d$ [MW]
$D_{d,t}^{HD}$	Hour- $t$ heat demand at heat node $d$ [MW]
$GS_s^{\max/\min}$	Max/min gas stock in gas storage $s$ [MWh]
$HS_h^{\max/\min}$	Max/min heat reserve in heat storage $h$ [MWh]
$H_h^{HS,in/out,\max}$	Heat input/output capacity of heat storage $h$ [MW]
$H_j^{CHP,\max/\min}$	Max/min heat supply of CHP unit $j$ [MW]
$k_{mn}$	Heat transfer coefficient of pipeline $m-n$
$LP_l^{\max/\min}$	Max/min gas linepack in pipeline $l$ [MWh]
$m_{mn,t}^{\max/\min}$	Max/min water flow rate in water pipeline $m-n$ [m <sup>3</sup> /h]
$P_i^{CFP,\max/\min}$	Max/min power generation level of CFP unit $i$ [MW]
$P_j^{CHP,\max/\min}$	Max/min power generation level of CHP unit $j$ [MW]
$P_{nm}^{\max}$	The capacity of transmission line $n-m$ [MW]
$Q_{g,t}^{GS,\max/\min}$	Max/min gas supply level of gas source $g$ [MW]
$Q_k^{P2G,\max/\min}$	Max/min gas generation level of P2G unit $k$ [MW]
$Q_{s,t}^{ST,\max/\min}$	Gas input/output capacity of gas storage $s$ [MW]
$S_{nm}^{\max}$	Transmission capacity of gas pipeline $n-m$ [MW]
$\tau_n^{\max/\min}$	Max/min nodal temperature [°C]
$\tau_{mn}^{\text{in/out},\max/\min}$	Max/min temperature at the inlet/outlet of pipe $m-n$ [°C]
$\sigma_{e,t}^{UE}$	Cost of unserved electrical energy [\$/MWh]
$W_{f,t}$	Power output of wind unit $f$ [MW]
$\lambda_g^{GC}$	Energy consumption coefficient of gas compressor $g$
$\eta_j^e$	Generating coefficient of CHP unit $j$
$\eta_j^l$	Heat loss coefficient of CHP unit $j$

$K^e$	Heat exchange coefficient of CHP unit $j$
$\eta_k^{P2G}$	The energy conversion efficiency of gas generation at P2G unit $k$
$\eta^{GC}$	Compression efficiency
$K^{GC}$	Constant of compressor
$E^{GC}$	Compressors parasitic efficiency
$T_s$	Suction temperature of compressor
$c_k$	Specific heat ratio for natural gas
$Z_a$	Average compressibility factor
$Z_{nm}$	The resistance coefficient of the gas pipeline
$CR$	Compression ratio

### Variables

$D_{j,t}^{CHP}$	Hour- $t$ gas consumption in CHP unit $j$ [MW]
$D_{g,t}^{GC}$	Gas consumption in gas compressor $g$ [MW]
$D_{h,t}^{HP,E}$	Power consumption in heat pump $h$ [MW]
$D_{k,t}^{P2G}$	Power consumption in P2G unit $k$ [MW]
$GS_{s,t}$	Gas stocks in gas storage $s$ [MWh]
$HS_{h,t}$	Heat stocks in heat storage $h$ [MWh]
$H_{h,t}^{HS,in/out}$	Heat input/output of heat storage $h$ [MW]
$H_{j,t}^{CHP}$	Heat supply from CHP unit $j$ [MW]
$H_{h,t}^{HP}$	Heat supply from HP $h$ [MW]
$LP_{l,t}$	Linepack stock in pipeline $l$ [MW]
$m_{mn,t}$	Water flow rate in water pipeline $m-n$ [m <sup>3</sup> /h]
$P_{i,t}^{CFP}$	Power production from CFP unit $i$ [MW]
$P_{j,t}^{CHP}$	Power supply from CHP unit $j$ [MW]
$P_{e,t}^{UE}$	Unserved electrical energy [MW]
$Q_{k,t}^{P2G}$	Gas supply from P2G unit $k$ [MW]
$Q_{g,t}^{GS}$	Gas supply from gas source $g$ [MW]

$Q_{s,t}^{ST,in/out}$	Gas input/output of gas storage $s$ [MW]
$Q_{l,t}^{LP,in/out}$	Gas input/output of linepack pipeline $l$ [MW]
$S_{mn,t}$	Gas flow rate in pipeline $m-n$ [MW]
$S_{g,t}^{GC}$	Gas flow rate through gas compressor $g$ [MW]
$W_{f,t}^{spill}$	Wind power spillage for wind unit $f$ [MW]
$\delta_{n,t}$	Phase angle of bus $n$ [rad]
$\pi_{n,t}$	Square of gas pressure of node $n$ [MPa <sup>2</sup> ]
$\tau_{n,t}$	Nodal temperature of water [°C]
$\tau_{mn,t}^{in/out}$	Water temperature of inlet/outlet of pipeline $m-n$ [°C]

We consider a multi-energy system controlled by a central entity who jointly operate the electricity, gas and district heating subsystems pursuing minimum cost. Therefore, the centralized model below is formulated to represent the joint scheduling of the electricity, gas and heating subsystems:

$$\begin{aligned} \min_x \sum_{t=1}^{n^T} \left\{ \sum_{i=1}^{n^{CFP}} C_i^{CFP} P_{i,t}^{CFP} + \sum_{g=1}^{n^{GS}} C_g^{GS} Q_{g,t}^{GS} \right. \\ \left. + \sum_{s=1}^{n^{ST}} \left( C_s^{ST,in} Q_{s,t}^{ST,in} + C_s^{ST,out} Q_{s,t}^{ST,out} \right) \right. \\ \left. + \sum_{h=1}^{n^{HS}} \left( C_h^{HS,in} H_{h,t}^{HS,in} + C_h^{HS,out} H_{h,t}^{HS,out} \right) \right\} \end{aligned} \quad (1)$$

subject to:

Linking constraints (2)-(5),

Operation constraints of the EPS (6)-(16),

Operation constraints of the NGS (17)-(32),

Operation constraints of the DHS (33)-(47).

## 2.2 Linkages among power, gas, and heat subsystems

The linkages among the electricity, gas, and district heating subsystems are the energy conversion relationships of the HPs (2), CHP units (3)-(4) and the P2G units (5).



Since HPs are more effective at heat generation than electrical resistance heaters, HP technologies are widely used for heating supply. The term coefficient of performance ( $COP$ ) is used to describe the ratio of useful heat generation per energy input:

$$H_{h,t}^{\text{HP}} = COP_h \times D_{h,t}^{\text{HP,E}}, \quad \forall h \in \Omega^{\text{HP}}, \forall t \in T \quad (2)$$

The gas-fired CHP units generate electricity and heat simultaneously by consuming natural gas. The relationship between the output of electricity and heat is given by (3), and the gas consumption by (4).

$$H_{j,t}^{\text{CHP}} = P_{j,t}^{\text{CHP}} \cdot (1 - \eta_j^e - \eta_j^h) / \eta_j^e \times K^e, \quad \forall j \in \Omega^{\text{CHP}}, \forall t \in T \quad (3)$$

$$D_{j,t}^{\text{CHP}} = P_{j,t}^{\text{CHP}} / \eta_j^e, \quad \forall j \in \Omega^{\text{CHP}}, \forall t \in T \quad (4)$$

P2G units consume electricity to produce gas. Equation (4) provides the relationship between the amount of gas production and the amount of consumed electricity.

$$Q_{k,t}^{\text{P2G}} = \eta_k^{\text{P2G}} D_{k,t}^{\text{P2G}}, \quad \forall k \in \Omega^{\text{P2G}}, \forall t \in T \quad (5)$$

### 2.3 Operation constraints of the EPS

$$\begin{aligned} & \sum_{j \in \Omega_n^{\text{CHP}}} P_{j,t}^{\text{CHP}} + \sum_{i \in \Omega_n^{\text{CFP}}} P_{i,t}^{\text{CFP}} + \sum_{f \in \Omega_n^{\text{WF}}} (W_{f,t} - W_{f,t}^{\text{spill}}) \\ & - \sum_{k \in \Omega_n^{\text{P2G}}} D_{k,t}^{\text{P2G,E}} - \sum_{h \in \Omega_n^{\text{HP}}} D_{h,t}^{\text{HP,E}} - \sum_{e \in \Omega_n^{\text{ED}}} (D_{e,t}^{\text{ED}} - P_{e,t}^{\text{UE}}) \\ & = \sum_{m \in \Lambda_n} B_{nm} (\delta_{n,t} - \delta_{m,t}), \quad \forall t \in T, \forall n \in \Lambda^{\text{EPS}} \end{aligned} \quad (6)$$

$$\delta_{\text{REF},t} = 0, \quad \forall t \in T \quad (7)$$

$$P_i^{\text{CFP},\min} \leq P_{i,t}^{\text{CFP}} \leq P_i^{\text{CFP},\max}, \quad \forall i \in \Omega^{\text{CFP}}, \forall t \in T \quad (8)$$

$$P_j^{\text{CHP},\min} \leq P_{j,t}^{\text{CHP}} \leq P_j^{\text{CHP},\max}, \quad \forall j \in \Omega^{\text{CHP}}, \forall t \in T \quad (9)$$

$$P_{i,t}^{\text{CFP}} - P_{i,t-1}^{\text{CFP}} \leq RU_i^{\text{CFP}}, \quad \forall i \in \Omega^{\text{CFP}}, \forall t \in T \quad (10)$$

$$P_{i,t-1}^{\text{CFP}} - P_{i,t}^{\text{CFP}} \leq RD_i^{\text{CFP}}, \quad \forall i \in \Omega^{\text{CFP}}, \forall t \in T \quad (11)$$

$$P_{j,t}^{\text{CHP}} - P_{j,t-1}^{\text{CHP}} \leq RU_j^{\text{CHP}}, \quad \forall j \in \Omega^{\text{CHP}}, \forall t \in T \quad (12)$$

$$P_{j,t-1}^{\text{CHP}} - P_{j,t}^{\text{CHP}} \leq RD_j^{\text{CHP}}, \quad \forall j \in \Omega^{\text{CHP}}, \forall t \in T \quad (13)$$

$$-P_{nm}^{\max} \leq B_{nm} (\delta_{n,t} - \delta_{m,t}) \leq P_{nm}^{\max}, \quad \forall n, m \in \Lambda^{\text{EPS}}, \forall t \in T \quad (14)$$

$$0 \leq W_{f,t}^{\text{spill}} \leq W_{f,t}, \quad \forall f \in \Omega^{\text{WF}}, \forall t \in T \quad (15)$$

$$0 \leq P_{e,t}^{\text{UE}} \leq D_{e,t}^{\text{ED}}, \quad \forall e \in \Omega^{\text{ED}}, \forall t \in T \quad (16)$$

The operational constraints of the power system (6)-(16), which consist of the nodal power balance equation (6), the constraints (7) that fix the phase angle at the reference node to zero, capacities of CFP (8) and CHP (9) units, ramping limits of CFP (10)-(11) and CHP (12)-(13) units,

and power transmission capacities (14). Additionally, constraints (15) and (16) enforce that the amount of curtailed wind power and unserved electrical energy should be lower than or equal to the wind power output and power load, respectively.

## 2.4 Operation constraints of the NGS

$$\pi_{n,t} - \pi_{m,t} = Z_{nm} (S_{nm,t})^2, \quad \forall n, m \in \Lambda^{\text{NGS}}, \forall t \in T \quad (17)$$

$$\begin{aligned} & \sum_{g \in \Omega_n^{\text{GS}}} Q_{g,t}^{\text{GS}} + \sum_{s \in \Omega_n^{\text{ST}}} (Q_{s,t}^{\text{ST,out}} - Q_{s,t}^{\text{ST,in}}) + \sum_{k \in \Omega_n^{\text{P2G}}} Q_{k,t}^{\text{P2G}} \\ & + \sum_{l \in \Omega_n^{\text{LP}}} (Q_{l,t}^{\text{LP,out}} - Q_{l,t}^{\text{LP,in}}) - \sum_{d \in \Omega_n^{\text{GD}}} D_{d,t}^{\text{GD}} - \sum_{g \in \Omega_n^{\text{GC}}} D_{g,t}^{\text{GC}} \\ & - \sum_{j \in \Omega_n^{\text{CHP}}} D_{j,t}^{\text{CHP}} = \sum_{m \in \Lambda_n} S_{nm,t}, \quad \forall m, n \in \Lambda^{\text{NGS}}, \forall t \in T \end{aligned} \quad (18)$$

$$\pi_{\text{REF},t} = 1, \quad \forall t \in T \quad (19)$$

$$D_{g,t}^{\text{GC}} = \lambda_g^{\text{GC}} S_{g,t}^{\text{GC}}, \quad \forall g \in \Omega^{\text{GC}}, \forall t \in T \quad (20)$$

$$ST_{s,t} = ST_{s,0} + \sum_{\tau=1}^t (Q_{s,\tau}^{\text{ST,in}} - Q_{s,\tau}^{\text{ST,out}}), \quad \forall s \in \Omega^{\text{ST}}, \forall t \in T \quad (21)$$

$$LP_{l,t} = LP_{l,0} + \sum_{\tau=1}^t (Q_{l,\tau}^{\text{LP,in}} - Q_{l,\tau}^{\text{LP,out}}), \quad \forall l \in \Omega^{\text{LP}}, \forall t \in T \quad (22)$$

$$LP_{l,n^T} = LP_{l,0}, \quad \forall l \in \Omega^{\text{LP}} \quad (23)$$

$$Q_g^{\text{GS,min}} \leq Q_{g,t}^{\text{GS}} \leq Q_g^{\text{GS,max}}, \quad \forall g \in \Omega^{\text{GS}}, \forall t \in T \quad (24)$$

$$Q_k^{\text{P2G,min}} \leq Q_{k,t}^{\text{P2G}} \leq Q_k^{\text{P2G,max}}, \quad \forall k \in \Omega^{\text{P2G}}, \forall t \in T \quad (25)$$

$$0 \leq Q_{s,t}^{\text{ST,in}} \leq Q_s^{\text{ST,in,max}}, \quad \forall s \in \Omega^{\text{ST}}, \forall t \in T \quad (26)$$

$$0 \leq Q_{s,t}^{\text{ST,out}} \leq Q_s^{\text{ST,out,max}}, \quad \forall s \in \Omega^{\text{ST}}, \forall t \in T \quad (27)$$

$$\pi_{n,t}^{\text{min}} \leq \pi_{n,t} \leq \pi_{n,t}^{\text{max}}, \quad \forall n \in \Lambda^{\text{NGS}}, \forall t \in T \quad (28)$$

$$-S_{nm}^{\text{max}} \leq S_{nm,t} \leq S_{nm}^{\text{max}}, \quad \forall m, n \in \Lambda^{\text{NGS}}, \forall t \in T \quad (29)$$

$$ST_s^{\text{min}} \leq ST_{s,t} \leq ST_s^{\text{max}}, \quad \forall s \in \Omega^{\text{ST}}, \forall t \in T \quad (30)$$

$$LP_l^{\text{min}} \leq LP_{l,t} \leq LP_l^{\text{max}}, \quad \forall l \in \Omega^{\text{LP}}, \forall t \in T \quad (31)$$

The network constraints include the pipeline flow equations (17) and the nodal gas balance equations (18). Constraint (19) fixes the gas pressure at the reference node to 1MPa in each hour. Constraint (20) provides the energy consumption in gas compressors (GC), where  $\lambda_g^{\text{GC}}$  is an operational parameter of any gas compressor and the compression ratio ( $CR$ ) is given by (33). For simplicity, we assume that gas compressors work with a constant compression ratio:

$$\lambda_g^{\text{GC}} = K^{\text{GC}} Z_a \left[ \frac{T_s}{E^{\text{GC}} \eta^{\text{GC}}} \right] \left[ \frac{c_k}{c_k - 1} \right] \left[ (CR)^{\frac{c_k - 1}{c_k}} - 1 \right] \quad (32)$$

Equations (21) and (22) enforce the temporal balance of gas storage and gas linepack, respectively. Equation (23) provides the linepack restoration constraints. Additionally, all facilities, including gas sources (24), P2G units (25), and gas storages (26)-(27) are subject to their operational limits. The gas pressure (28) and mass flow rate (29) should be within their respective operational bounds. Equations (30) and (31) enforce that gas storages and gas linepacks should operate within safety margins.

## 2.5 Optimal operation of the DHS

$$\begin{aligned} & \sum_{j \in \Omega_n^{\text{CHP}}} H_{j,t}^{\text{CHP}} + \sum_{k \in \Omega_n^{\text{P2G}}} H_{k,t}^{\text{P2G}} + \sum_{s \in \Omega_n^{\text{HS}}} (H_{s,t}^{\text{HS,out}} - H_{s,t}^{\text{HS,in}}) \\ & + \sum_{h \in \Omega_n^{\text{HP}}} H_{h,t}^{\text{HP}} - \sum_{l \in \Omega_n^{\text{HL}}} H_{l,t}^{\text{HL}} = c \cdot m_{n,t} \cdot (\tau_{n,t}^{\text{out}} - \tau_{n,t}^{\text{in}}), \quad (33) \\ & \forall n \in \Lambda^{\text{DHS}}, \forall t \in T \end{aligned}$$

$$\tau_{mn,t}^{\text{out}} - \tau_{mn,t}^{\text{am}} = \lambda_{mn,t} (\tau_{mn,t}^{\text{in}} - \tau_{mn,t}^{\text{am}}), \forall m, n \in \Lambda^{\text{DHS}}, \forall t \in T \quad (34)$$

$$\tau_{n,t} \sum_{m \in \Lambda_n} m_{mn,t} = \sum_{m \in \Lambda_n} (m_{mn,t} \cdot \tau_{mn,t}^{\text{out}}), \forall m, n \in \Lambda^{\text{DHS}}, \forall t \in T \quad (35)$$

$$\tau_{\text{REF},t} = 90, \quad \forall t \in T \quad (36)$$

$$\sum_{m \in \Lambda_n} m_{mn,t} = 0, \forall m, n \in \Lambda^{\text{DHS}}, \forall t \in T \quad (37)$$

$$H_j^{\text{CHP,min}} \leq H_{j,t}^{\text{CHP}} \leq H_j^{\text{CHP,max}}, \quad \forall j \in \Omega^{\text{CHP}}, \forall t \in T \quad (38)$$

$$H_p^{\text{HP,min}} \leq H_{p,t}^{\text{HP}} \leq H_p^{\text{HP,max}}, \quad \forall p \in \Omega^{\text{HP}}, \forall t \in T \quad (39)$$

$$HS_{h,t+1} = HS_{h,t} + (H_{h,t}^{\text{HS,in}} - H_{h,t}^{\text{HS,out}}), \quad \forall h \in \Omega^{\text{HS}}, \forall t \in T \quad (40)$$

$$H_h^{\text{HS,in,min}} \leq H_{h,t}^{\text{HS,in}} \leq H_h^{\text{HS,in,max}}, \quad \forall h \in \Omega^{\text{HS}}, \forall t \in T \quad (41)$$

$$H_h^{\text{HS,out,min}} \leq H_{h,t}^{\text{HS,out}} \leq H_h^{\text{HS,out,max}}, \quad \forall h \in \Omega^{\text{HS}}, \forall t \in T \quad (42)$$

$$HS_h^{\text{min}} \leq HS_{h,t} \leq HS_h^{\text{max}}, \quad \forall h \in \Omega^{\text{HS}}, \forall t \in T \quad (43)$$

$$m_{mn}^{\text{min}} \leq m_{mn,t} \leq m_{mn,t}^{\text{max}}, \forall m, n \in \Lambda^{\text{DHS}}, \forall t \in T \quad (44)$$

$$\tau_n^{\text{min}} \leq \tau_{n,t} \leq \tau_n^{\text{max}}, \forall m, n \in \Lambda^{\text{DHS}}, \forall t \in T \quad (45)$$

$$\tau_{mn}^{\text{in,min}} \leq \tau_{mn,t}^{\text{in}} \leq \tau_{mn}^{\text{in,max}}, \forall m, n \in \Lambda^{\text{DHS}}, \forall t \in T \quad (46)$$

$$\tau_{mn}^{\text{out,min}} \leq \tau_{mn,t}^{\text{out}} \leq \tau_{mn}^{\text{out,max}}, \forall m, n \in \Lambda^{\text{DHS}}, \forall t \in T \quad (47)$$

The DHS is a double pipeline network, including supply and return networks, which consists of heat sources, heat storages, heat pipelines, and heat loads. Constraint (33) states the nodal heat balance. The left-hand side is the total heat supplied by all units connected to node  $n$  minus the heat load at that node in hour  $t$ . The right-hand side gives the temperature increase due to the heat exchange. Equation (34) gives the temperature drop along the water pipeline, which is the major energy loss in the DHS.  $\lambda_{mn,t}$  is an operational parameter of the water pipeline related to

the mass flow rate ( $m_{mn,t}$ ) as shown below.

$$\lambda_{mn,t} = \exp\left(-\frac{k_{mn}\pi d_{mn}l_{mn}}{c \cdot m_{mn,t}}\right) \quad (48)$$

Constraint (35) gives the nodal temperature of the mixed water on the basis of energy conservation. Constraint (36) fixes the water temperature at the reference node to 90 degrees centigrade. Constraint (37) enforces the nodal mass flow balance. Constraints (38) and (39) enforce the heat production capacities of CHP units and HPs, respectively. Constraints (40) enforces the temporal balance of heat storage. Constraints (41)-(43) enforce that heat storage is subject to their operational limits. The water transmission capacities are given in (44). Finally, constraints (45)-(47) state that the water temperatures should be within the operational limits.

### 3. Case study

Fig. 1 shows a test system considered in this work. This test system includes an electricity system, a heating system and a gas network. The electricity system is composed of a CHP unit, a CFP unit, and a wind farm. The gas network includes a gas source, a gas storage and a gas compressor. The heating system includes heat storage and three nodes. There are two links: CHP.

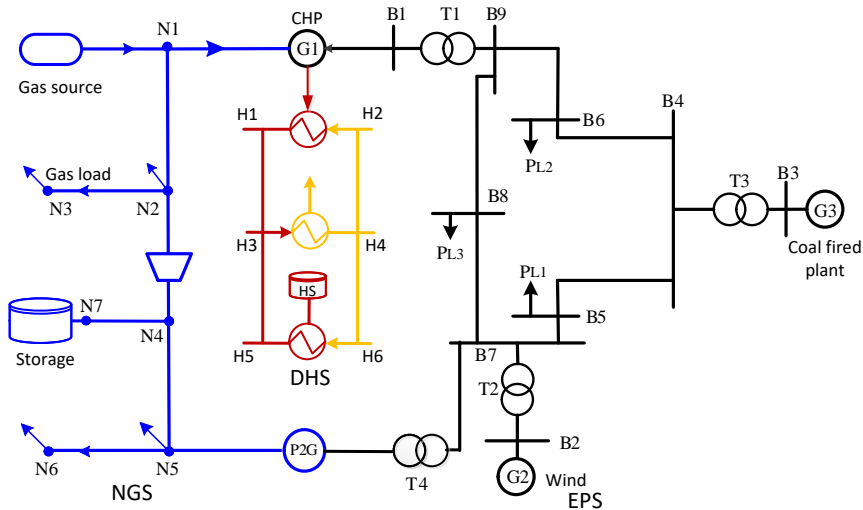


Fig. 1 Time series of renewable power generation and inflexible load.

### 3.1 Network operational parameters

Just as the voltage stability, which plays a major role in the electrical power system, the gas pressure and water temperature are critical factors for the security operation of natural gas system and district heating system, respectively. Fig. 2 shows the water temperature at the inlet and outlet of node 2, which is a load node. The reference temperature is fixed to 80 °C. As there are heat losses generated during the process of heat transmission, the water temperatures at the inlet of node 2 are lower than the reference node. Besides, there is a lot of heat exchange happen at the load node. Thus the water temperature at the outlet of node 2 will be further declined. The level of temperature drop at load node mainly depends on the amount of heat load. It shows that the temperature drop is larger during the night hours when there is higher heat demand. Finally, due to heat supply from P2G unit, the inlet and outlet temperatures of integrated system with P2G are higher than that without P2G. A higher outlet temperature helps to guarantee safety operation of heating network.

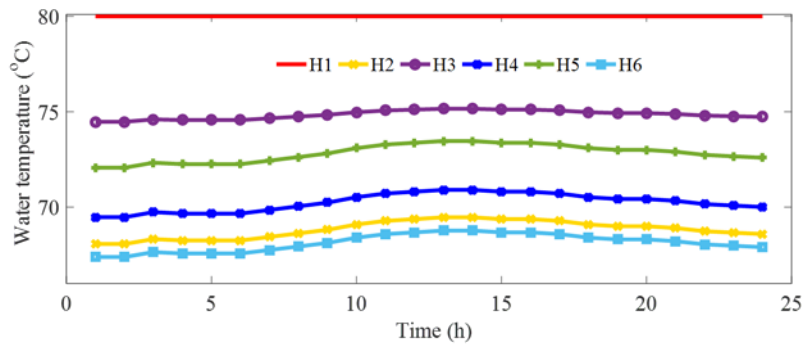


Fig. 2 Time series of net load.

Fig. 3 shows the variability of nodal gas pressure. The reference pressure is fixed to 1 bar. The nodal gas pressure varies in a narrow range, which indicates that the gas system can play a stabilizing role in multi-energy systems as there is gas linepack in the operational process. It is illustrated that both the district heating system and the natural gas system can provide flexibility to accommodate the fluctuation of the electrical power system.

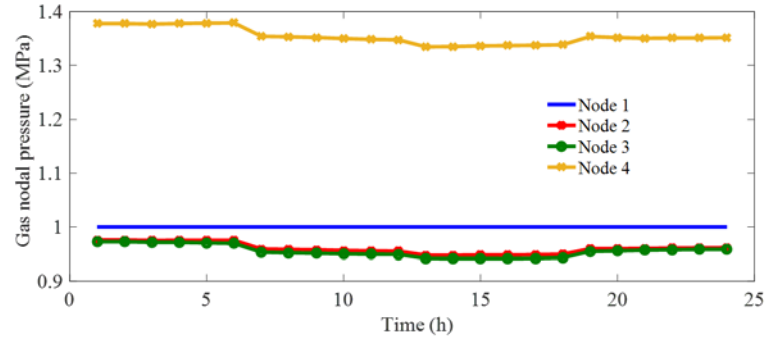


Fig. 3 Ramp needs of electricity load.

### 3.2 Scheduling of the energy sources

This subsection analyzes the scheduling strategy throughout a 24-hour time horizon.

Fig. 4 illustrates the optimal schedule of the electrical system. The total electricity consumption is composed of the power load and the electricity consumption at the P2G unit. In this case, there is a high wind power output, but only a small amount of surplus wind power is curtailed in the night. Most of the excess electricity is converted into gas and heat by the P2G unit, which helps to reduce the wind curtailment. The total power load includes both the nodal power load and electricity consumed by P2G. During the peak load periods (7:00 AM~20:00 PM), wind power is utilized to supply the electrical load. The CPG increase their unit outputs to balance the power demand. During the night (21:00 PM~24:00 PM), the excess wind power is converted into gas fuel by P2G.

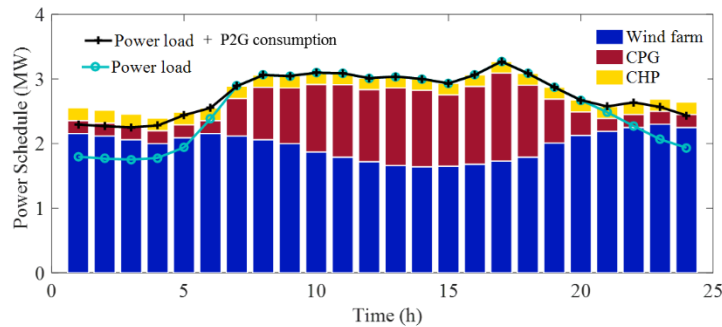


Fig. 4. Optimal schedule of the electrical power system

Fig. 5 illustrates the optimal schedule of the gas system. The total gas consumption includes both the gas demand and the gas consumption at CHP unit. The difference between gas production and consumption is balanced by linepack storage, which provides the flexibility to gas networks. Note that the gas supply from the gas well is flat. During the night, the excess wind power is converted into gas fuel by P2G and injected into gas network. The difference between gas supply and consumption is offset by the gas linepack.

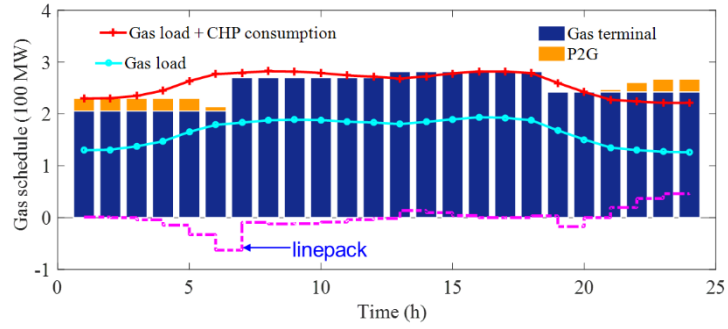


Fig. 5. Optimal schedule of the natural gas system

Fig. 6 illustrates the optimal schedule of district heating system. The total heat consumption includes the heat load and the heat loss. The heat loss comes from the heat dissipation of the high-temperature water. The daily heat loss is about 17.1% in this study. The total heat consumption includes both the heat load and the heat loss. The heat loss is mainly caused by the heat transfer from the high temperature water to the ambience.

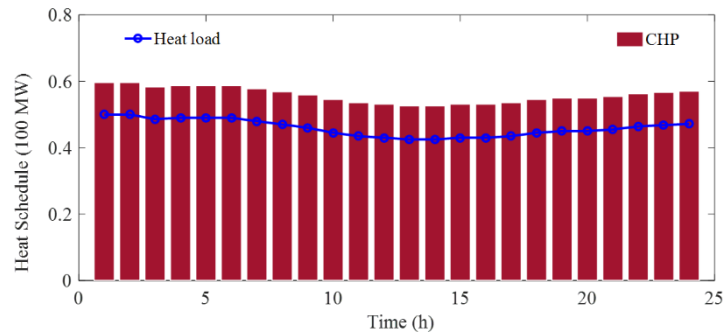


Fig. 6. Optimal schedule of the district heating system

## 4. Summary

We propose a coordinated optimization model for the joint operation of the electricity, gas, and district heating systems of an urban area. A nonlinear problem is formulated by considering the network constraints in the integrated systems. This model is solved using IPOPT under GAMS. Simulation results demonstrate the effectiveness of the proposed approach. The required computational time is acceptable with operational requirements. It shows that most of the surplus wind power can be converted into gas and heat by the P2G unit, which helps to reduce wind curtailment. Further, both the district heating system and the natural gas system can provide flexibility to accommodate the fluctuation of the electrical power system.



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