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Data-Driven Affine Policies for Flexibility Provision by Natural Gas Networks to Power Systems

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***Abstract*—Using flexibility from the coordination of power and natural gas systems helps with the integration of variable renewable energy in power systems. To include this flexibility into the operational decision-making problem, we propose a data-driven distributionally robust chance-constrained co-optimization of power and natural gas systems considering flexibility from short-term gas storage in pipelines, i.e., linepack. Recourse actions in both systems, based on linear decision rules, allow adjustments to the dispatch and operating set-points during real-time operation when the uncertainty in wind power production is revealed. We convexify the non-linear and non-convex power and gas flow equations using DC power flow approximation and second-order cone relaxation, respectively. Our coordination approach enables a study of the mitigation of short-term uncertainty propagated from the power system to the gas side. We analyze the results of the proposed approach on a case study and evaluate the solution quality via out-of-sample simulations performed *ex-ante*.**

***Index Terms*—Data-driven linear decision rules, Distributionally robust chance constraints, Linepack flexibility, Power and natural gas coordination, Second-order cone program.**

I. Introduction

Natural gas-fired power plants (NGFPPs) typically provide operational flexibility to power systems with a high share of intermittent renewable energy. Short-term gas storage in natural gas pipelines, known as linepack, provides an additional source of flexibility [1] at no extra investment cost. Efficient procurement of flexibility from the natural gas system during day-ahead scheduling of power systems requires consideration of the operational constraints of the natural gas system. Further, with the increasing share of intermittent renewable energy sources in the power system, the need for flexibility and thereby, the interdependence between the power and natural gas system is becoming stronger [2]. As a result, the coordination between power and natural gas systems during the day-ahead dispatch has been a topic of research interest in recent years. Various levels of coordination and information exchange between the systems are discussed in [3], [4], [5], while [6], [7], [8], [9], [10], [11] model full integration of the power with the natural gas system. The value of gas system related flexibility for the power system is quantified in [8] and [9] in a deterministic manner.

Increasing interactions between power and natural gas systems, however, result in the propagation of short-term uncertainty faced by power systems to the gas side. Prior works on the coordinated operation of power and natural gas systems have largely ignored this short-term uncertainty. This may result in additional recourse actions necessary during the real-time operation stage when the flexibility from the natural gas system is not correctly anticipated. Affine policies, built on the theory of linear decision rules, have been a preferred choice for day-ahead decision making, wherein nominal dispatch schedules along with the recourse actions for real-time operation are optimally decided [12]. In this paper, we introduce a unified framework to elicit flexibility based on affine policies from agent assets, e.g., generators, natural gas suppliers as well as the network assets, i.e., linepack. Our data-driven affine policies are decided based on the features of uncertainty drawn from the historical measurements, with no distributional restriction imposed on the random variables.

Previous works discussing uncertainty-aware coordination between power and natural gas systems use stochastic programming approaches such as scenario-based [7], robust [10], and chance-constrained optimization [11]. Reference [7] proposes a two-stage stochastic program for the day-ahead and real-time operations of integrated power and natural gas system under uncertainty from renewable generation. In a similar direction, an adjustable robust dispatch framework is proposed in [10]. Chance-constraints are introduced into the planning problem of the integrated power and natural gas system [11].

While scenario-based approaches [7] incur a high computational expense due to a large number of scenarios needed to characterize the uncertainty properly, robust approaches [10] often suffer from over-conservativeness of the solution due to the design objective to minimize *worst-case* cost. Distributionally robust chance-constrained formulation of the problem allows for an adjustable probabilistic violation of operational limits when facing extreme realizations of uncertainty, characterized by an ambiguity set.

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We adopt a data-driven distributionally robust chance-constrained optimization technique, considering its advantages over other stochastic programming approaches, to introduce a coordinated day-ahead dispatch of power and natural gas systems taking the flexibility provided by linepack into account. Our main contribution is a tractable reformulation of distributionally robust chance constraints for the combined dispatch problem considering linepack. Thereby, we study the propagation of short-term uncertainty to the natural gas system and evaluate the potential of the natural gas network as flexibility provider. This could potentially result in the design of new market-based coordination mechanisms and market products enabling gas system agents and the network to play an active role in providing flexibility to power systems.

The rest of this paper is organized as follows: Section II presents the distributionally robust chance-constrained power and natural gas dispatch problem. Section III discusses the solution methodology, which is then applied to a case study in Section IV. Finally, conclusions are drawn and the scope for future work is discussed in Section V.

II. Problem Formulation

A. Preliminaries

In the following, we introduce the operation of a coupled power and natural gas system, wherein power generated from dispatchable power plants $i \in \mathcal{I}$ and wind farms $j \in \mathcal{J}$ is used to meet the inelastic electricity demand from a set of loads $d \in \mathcal{D}$. The dispatchable generators comprise of NGFPPs $g \in \mathcal{G} \subset \mathcal{I}$ and non-NGFPPs $c \in \mathcal{C} \subset \mathcal{I}$. On the gas side, natural gas suppliers $k \in \mathcal{K}$, together with available linepack in the gas network, are dispatched to meet the natural gas demand from inelastic gas loads and the fuel needed by NGFPPs. We assume that wind power is available at zero marginal cost of production and that excess generation from wind farms can be spilled at no cost. Power produced by wind farms during real-time operation is considered as the only source of uncertainty.

B. Uncertainty Model

For wind farm j, the day-ahead point forecast for time period $t \in \mathcal{T}$ is given by $W^{\text{PF}}_{j,t}$. The forecast error that occurs in real-time is assumed to be a random variable $\delta_{j,t}$, such that the overall system uncertainty can be characterized by $\Omega = [\delta_{11}\ \delta_{21}\ \dots\ \delta_{|\mathcal{J}|t}\ \dots\ \delta_{|\mathcal{J}||\mathcal{T}|}]^\top \in \mathbb{R}^{|\mathcal{J}||\mathcal{T}|}$, where $\mathbb{R}$ is the set of real numbers and $|\cdot|$ is the cardinality operator over a set. We consider that Ω follows an unknown multivariate probability distribution $\mathbb{P} \in \Pi$, where Π is an ambiguity set defined as

$$\Pi = \{\mathbb{P} \in \Pi_0(\mathbb{R}^{|\mathcal{J}|}) : \mathbb{E}_{\mathbb{P}}[\Omega] = \boldsymbol{\mu}^{\Pi}, \mathbb{E}_{\mathbb{P}}[\Omega^\top \Omega] = \boldsymbol{\Sigma}^{\Pi}\}, \quad (1)$$

such that the family of distributions, $\Pi_0(\mathbb{R}^{|\mathcal{J}|})$ contains all probability distributions whose first and second-order moments are given by known parameters $\boldsymbol{\mu}^{\Pi} \in \mathbb{R}^{|\mathcal{J}||\mathcal{T}|}$ and $\boldsymbol{\Sigma}^{\Pi} \in \mathbb{R}^{|\mathcal{J}||\mathcal{T}| \times |\mathcal{J}||\mathcal{T}|}$, respectively and $(\cdot)^\top$ is the transpose operator. Without any loss of generality, we assume that the mean $\boldsymbol{\mu}^{\Pi} = 0$ and that the covariance matrix $\boldsymbol{\Sigma}^{\Pi}$ can be empirically estimated from historical record of wind forecast errors. The structure of the positive semi-definite covariance matrix, $\boldsymbol{\Sigma}^{\Pi}$ is such that its diagonal blocks, comprised of sub-matrices, $\boldsymbol{\Sigma}^{\Pi}_t \in \mathbb{R}^{|\mathcal{J}| \times |\mathcal{J}|}, \forall t \in \mathcal{T}$, capture the spatial correlation among the wind forecast errors in period t, while the off-diagonal blocks contain information about spatio-temporal correlation of the uncertain parameters.

With this description of uncertain wind forecast errors, the net error or net deviation from the point forecasts of all wind farms in the time period t is $\mathbf{e}^\top \Omega_t$ where $\mathbf{e} \in \mathbb{R}^{|\mathcal{J}|}$ is a vector of all ones. The temporally collapsed random vector is formed as: $\Omega_t = F_t \Omega$, where $F_t \in \mathbb{R}^{|\mathcal{J}| \times |\mathcal{J}||\mathcal{T}|}$ is a *selector matrix* formed by blocks of null matrices $\mathbf{0} \in \mathbb{R}^{|\mathcal{J}| \times |\mathcal{J}|}$ and a single block of identity matrix $\mathbf{1} \in \mathbb{R}^{|\mathcal{J}| \times |\mathcal{J}|}$, starting at column $(|\mathcal{J}|(t-1)+1),\ \forall t \in \mathcal{T}$. As a sign convention, $\mathbf{e}^\top \Omega_t > 0$ implies deficit of wind power available in the system during real-time operation stage as compared to the day-ahead forecast.

C. Uncertainty-aware Power and Natural Gas Coordination

The proposed day-ahead coordinated electricity and natural gas model is a stochastic program, presented in (2). The objective function has a min-max structure such that the total system dispatch cost is minimized while the uncertain variable Ω draws from a probability distribution $\mathbb{P} \in \Pi$ that results in maximizing the cost of dispatch, i.e., the worst-case probability distribution. The stochastic program is formulated by the following optimization problem:

$$\min_{\Theta_1} \max_{\mathbb{P} \in \Pi} \mathbb{E}_{\mathbb{P}}\left[\sum_{t \in \mathcal{T}} \Big(\sum_{i \in \mathcal{C}} C^{\text{E}}_i\ \tilde{p}_{i,t} + \sum_{k \in \mathcal{K}} C^{\text{G}}_k\ \tilde{g}_{k,t} \Big)\right] \quad (2a)$$

subject to

$$\mathbf{e}^\top \tilde{\mathbf{p}}_t + \mathbf{e}^\top (\mathbf{W}^{\text{PF}}_t - \Omega_t) = \mathbf{e}^\top \mathbf{D}^{\text{E}}_t,\ \forall t, \quad (2b)$$

$$\tilde{\mathbf{P}}^{\text{inj}}_t = \boldsymbol{\Psi}_{\text{I}} \tilde{\mathbf{p}}_t + \boldsymbol{\Psi}_{\text{J}} (\mathbf{W}^{\text{PF}}_t - \Omega_t) - \boldsymbol{\Psi}_{\text{D}} \mathbf{D}^{\text{E}}_t,\ \forall t, \quad (2c)$$

$$\min_{\mathbb{P} \in \Pi} \mathbb{P}[\{\boldsymbol{\Psi} \tilde{\mathbf{P}}^{\text{inj}}_t\}_{(n,r)} \geq -\{\overline{\mathbf{F}}\}_{(n,r)}] \geq (1 - \epsilon_{nr}),\ \forall (n,r) \in \mathcal{L},\ \forall t, \quad (2d)$$

$$\min_{\mathbb{P} \in \Pi} \mathbb{P}[\{\boldsymbol{\Psi} \tilde{\mathbf{P}}^{\text{inj}}_t\}_{(n,r)} \leq \{\overline{\mathbf{F}}\}_{(n,r)}] \geq (1 - \epsilon_{nr}),\ \forall (n,r) \in \mathcal{L},\ \forall t, \quad (2e)$$

$$\min_{\mathbb{P} \in \Pi} \mathbb{P}[\tilde{p}_{i,t} \geq \underline{P}_i] \geq (1 - \epsilon_i),\ \forall i,\ \forall t, \quad (2f)$$

$$\min_{\mathbb{P} \in \Pi} \mathbb{P}[\tilde{p}_{i,t} \leq \overline{P}_i] \geq (1 - \epsilon_i),\ \forall i,\ \forall t, \quad (2g)$$

$$\min_{\mathbb{P} \in \Pi} \mathbb{P}[\tilde{g}_{k,t} \geq \underline{G}_k] \geq (1 - \epsilon_k),\ \forall k,\ \forall t, \quad (2h)$$

$$\min_{\mathbb{P} \in \Pi} \mathbb{P}[\tilde{g}_{k,t} \leq \overline{G}_k] \geq (1 - \epsilon_k),\ \forall k,\ \forall t, \quad (2i)$$

$$\min_{\mathbb{P} \in \Pi} \mathbb{P}[\tilde{pr}_{m,t} \geq \underline{PR}_m] \geq (1 - \epsilon_m),\ \forall m,\ \forall t, \quad (2j)$$

$$\min_{\mathbb{P} \in \Pi} \mathbb{P}[\tilde{pr}_{m,t} \leq \overline{PR}_m] \geq (1 - \epsilon_m),\ \forall m,\ \forall t, \quad (2k)$$

$$\min_{\mathbb{P} \in \Pi} \mathbb{P}[\tilde{pr}_{u,t} \leq \Gamma_{m,u}\ \tilde{pr}_{m,t}] \geq (1 - \epsilon_{mu}),\ \forall (m,u) \in \mathcal{Z}_c,\ \forall t, \quad (2l)$$

$$\min_{\mathbb{P} \in \Pi} \mathbb{P}[\tilde{q}_{m,u,t} \geq 0] \geq (1 - \epsilon_{mu}),\ \forall (m,u) \in \mathcal{Z},\ \forall t, \quad (2m)$$

$$\min_{\mathbb{P} \in \Pi} \mathbb{P}[\tilde{q}^{\text{in}}_{m,u,t} \geq 0] \geq (1 - \epsilon_{mu}),\ \forall (m,u) \in \mathcal{Z},\ \forall t, \quad (2n)$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[\tilde{q}^{\text{out}}_{m,u,t} \geq 0] \geq (1-\epsilon_{mu}),\ \forall (m,u)\in\mathcal{Z},\ \forall t, \tag{2o}$$

$$\tilde{q}^2_{m,u,t} = K^2_{m,u}(\tilde{pr}^2_{m,t} - \tilde{pr}^2_{u,t}),\ \forall (m,u)\in\mathcal{Z},\ \forall t, \tag{2p}$$

$$\tilde{q}_{m,u,t} = \frac{\tilde{q}^{\text{in}}_{m,u,t} + \tilde{q}^{\text{out}}_{m,u,t}}{2},\ \forall (m,u)\in\mathcal{Z},\ \forall t, \tag{2q}$$

$$\tilde{h}_{m,u,t} = S_{m,u}\frac{\tilde{pr}_{m,t} + \tilde{pr}_{u,t}}{2},\ \forall (m,u)\in\mathcal{Z},\ \forall t, \tag{2r}$$

$$\tilde{h}_{m,u,t} = H^0_{m,u} + \tilde{q}^{\text{in}}_{m,u,t} - \tilde{q}^{\text{out}}_{m,u,t}, \forall (m,u)\in\mathcal{Z}, t=1, \tag{2s}$$

$$\tilde{h}_{m,u,t} = \tilde{h}_{m,u,(t-1)} + \tilde{q}^{\text{in}}_{m,u,t} - \tilde{q}^{\text{out}}_{m,u,t},\quad \forall (m,u)\in\mathcal{Z}, t>1, \tag{2t}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[\tilde{h}_{m,u,t} \geq H^0_{m,u}] \geq (1-\epsilon_{mu}),\quad \forall (m,u)\in\mathcal{Z}, t=|\mathcal{T}|, \tag{2u}$$

$$\sum_{k\in\mathcal{A}^{\text{K}}_m} \tilde{g}_{k,t} - \sum_{i\in\mathcal{A}^{\text{G}}_m} \phi_i\ \tilde{p}_{i,t} - \sum_{u:(m,u)\in\mathcal{Z}} (\tilde{q}^{\text{in}}_{m,u,t} - \tilde{q}^{\text{out}}_{u,m,t}) = D^{\text{G}}_{m,t},\ \forall m,\ \forall t, \tag{2v}$$

where the set of uncertainty-dependent stochastic variables is $\Theta_1 = \{\tilde{p}_{i,t},\ \tilde{g}_{k,t},\ \tilde{pr}_{m,t},\ \tilde{q}_{m,u,t},\ \tilde{q}^{\text{in}}_{m,u,t},\ \tilde{q}^{\text{out}}_{m,u,t},\ \tilde{h}_{m,u,t}\}$. The terms in objective (2a) are the expected cost of power generation by non-NGFPPs and the cost of natural gas supply by gas suppliers, respectively.

The inequalities (2d)-(2o) and (2u) are modeled as distributionally robust chance constraints. This means that at the optimal solution to problem (2), the probability of meeting each individual constraint inside the square brackets $\mathbb{P}[\cdot]$ is modeled to have a confidence level of at least $(1-\epsilon_{(\cdot)})$, where each $\epsilon_{(\cdot)} \in [0,1]$. Subscripts $(\cdot)$ take the appropriate indices from the set $\{i,\ (n,r),\ k,\ m,\ (m,u)\}$ depending on the individual constraint.

Constraints (2b)-(2g) pertain to the power system. These constraints include the power balance (2b), limits on the stochastic power flows in the transmission lines (2c)-(2e) and the upper ($\underline{P}_i$) and lower bounds ($\overline{P}_i$) on the stochastic power production of generators (2f) and (2g). Vectors $\tilde{\mathbf{p}}_t \in \mathbb{R}^{|\mathcal{I}|}$, $\mathbf{W}^{\text{PF}}_t \in \mathbb{R}^{|\mathcal{J}|}$ and $\mathbf{D}^{\text{E}}_t \in \mathbb{R}^{|\mathcal{D}|}$ represent the power produced by generators, wind forecasts for wind farms and electricity demand from loads in period t, while Ω_t is the random vector of forecast errors, as previously defined. Vector coefficients, $\mathbf{e}$ in (2b) are of appropriate dimensions such that the total supply and demand are balanced in each period t. The matrix $\mathbf{\Psi} \in \mathbb{R}^{|\mathcal{L}|\times|\mathcal{N}|}$ represents the Power Transfer Distribution Factor (PTDF) matrix, derived from the reactances of power transmission lines [13], which maps the injections $\tilde{\mathbf{P}}^{\text{inj}}_t \in \mathbb{R}^{|\mathcal{N}|}$ at the electricity nodes to the power flows in each of the power lines $(n,r)\in\mathcal{L}$ in the network. Similarly, matrices $\mathbf{\Psi}_{\text{I}} \in \mathbb{R}^{|\mathcal{N}|\times|\mathcal{I}|}$, $\mathbf{\Psi}_{\text{J}} \in \mathbb{R}^{|\mathcal{N}|\times|\mathcal{J}|}$, and $\mathbf{\Psi}_{\text{D}} \in \mathbb{R}^{|\mathcal{N}|\times|\mathcal{D}|}$ map generators, wind farms and loads to the electricity nodes, such that (2c) gives the nodal power injections for all the electricity nodes in the system.

Natural gas system constraints are given in (2h)-(2u). While constraints (2h) and (2i) limit the stochastic gas supply $\tilde{g}_{k,t}$ by supplier k in time period t to $\underline{G}_k$ and $\overline{G}_k$, (2j) and (2k) limit the nodal gas pressure $\tilde{pr}_{m,t}$ at each gas node $m\in\mathcal{M}$ to be within the physical limits $\underline{PR}_m$ and $\overline{PR}_m$. For the natural gas pipelines with compressors, $\mathcal{Z}_c \subset \mathcal{Z}$, compression is modeled linearly in (2l), which relate the inlet and outlet pressures of two adjacent nodes through compression factor $\Gamma_{m,u}$. We assume that the direction of gas flow in each pipeline $(m,u)\in\mathcal{Z}$ is predetermined and (2m)-(2o) enforce this flow direction in real-time. As remarked in reference [1], this assumption is non-limiting for gas transmission networks when considering day-ahead operational problems as opposed to a network expansion planning problem or that of a gas distribution system wherein injections from distributed gas producers (e.g., biogas plants) cannot be neglected. Equality constraints (2p), known as Weymouth equation, describe the flow $\tilde{q}_{m,u,t}$ (given as average of in- ($\tilde{q}^{\text{in}}_{m,u,t}$) and outflow ($\tilde{q}^{\text{out}}_{m,u,t}$) by (2q)) along pipeline (m,u) as a quadratic non-convex function of the pressures $\tilde{pr}_{m,t}$ and $\tilde{pr}_{u,t}$ at the inlet (m) and outlet (u) nodes of the pipeline scaled by the pipeline resistance constant $K_{m,u}$. Constraints (2r) define the amount of linepack in the pipelines as the average of inlet and outlet pressures, scaled by the pipeline parameter $S_{m,u}$. Following the modeling approach in [9], (2s)-(2u) describe the evolution of the amount of linepack $\tilde{h}_{m,u,t}$ in a pipeline over time, with constraints (2u) ensuring that the linepack is not depleted at the end of the simulation horizon beyond initial linepack amount $H^0_{m,u}$. Supply-demand balance of natural gas at each node is ensured in real-time by equality constraints (2v) which also couple the power and natural gas systems through the fuel consumed by the NGFPPs scaled by a fuel conversion factor ϕ_i. The sets $\mathcal{A}^{\text{K}}_m \subset \mathcal{K}$ and $\mathcal{A}^{\text{G}}_m \subset \mathcal{G}$ gather gas suppliers and NGFPPs that are located at node m, respectively, while $D^{\text{G}}_{m,t}$ is the nodal gas demand.

The requirement to solve the stochastic program (2) during the day-ahead stage renders the problem infinite dimensional, as the optimization variables are a function of uncertain parameters that are only revealed during real-time operation the day after. To enable solvability of the problem, we adopt recourse actions based on linear decision rules [14] to define affine control policies for the sources of flexibility in the coupled system, i.e., flexible power generation, natural gas supply and linepack. The assumption of affine response to uncertainty by flexible agents, although somewhat limiting in light of the non-linear dynamics of natural gas flow in the network, provides an intuitive understanding of the methodology behind uncertainty propagation from power systems to natural gas system at a lower complexity of exposition. Generalized decision rules, for instance as discussed in [15], is left for future work.

D. Affine Control Policies

When solving the day-ahead dispatch problem, faced with the uncertainty in wind forecast errors, sources of flexibility in the coupled system are assigned optimal affine policies, in addition to the nominal schedule, which govern their response to the realization of uncertainty during real-time operation.

a) Power Producers: The affine response from dispatchable power plants (NGFPPs and non-NGFPPs) is given by

$$\tilde{p}_{i,t} = p_{i,t} + (\mathbf{e}^\top\Omega_t)\alpha_{i,t},\ \forall i\in\mathcal{I}, \tag{3}$$

where $\tilde{p}_{i,t}$ is the stochastic power production of unit i in real-time, $p_{i,t}$ is the nominal power production schedule if the uncertainty were absent (perfect forecasts) and $\alpha_{i,t} \in [0,1]$ denotes the participation factor of the unit towards mitigation of the unplanned deviation.

b) Gas Suppliers: The stochastic natural gas supply by supplier k is given by

$$\tilde{g}_{k,t} = g_{k,t} + (\mathbf{e}^\top \Omega_t)\beta_{k,t},\ \forall k \in \mathcal{K}, \tag{4}$$

where $g_{k,t}$ is the nominal gas supply and $\beta_{k,t} \geq 0$ represents the participation factor of the supplier towards uncertainty mitigation.

c) Gas Network Operator: Under the assumption that natural gas pressure in the gas network is controllable at all gas nodes, we define the affine response by the gas network operator to regulate the amount of linepack as

$$\tilde{pr}_{m,t} = pr_{m,t} + (\mathbf{e}^\top \Omega_t)\rho_{m,t},\ \forall m,t, \tag{5}$$

where $pr_{m,t}$ and $\rho_{m,t} \geq 0$ denote the nominal pressure and the change in pressure at node m in response to the uncertainty, respectively. Similarly, the real-time natural gas flows in pipelines can be expressed as

$$\tilde{q}_{m,u,t} = q_{m,u,t} + (\mathbf{e}^\top \Omega_t)\gamma_{m,u,t},\ \forall (m,u) \in \mathcal{Z}, t, \tag{6a}$$
$$\tilde{q}^{\text{in}}_{m,u,t} = q^{\text{in}}_{m,u,t} + (\mathbf{e}^\top \Omega_t)\gamma^{\text{in}}_{m,u,t},\ \forall (m,u) \in \mathcal{Z}, t, \tag{6b}$$
$$\tilde{q}^{\text{out}}_{m,u,t} = q^{\text{out}}_{m,u,t} + (\mathbf{e}^\top \Omega_t)\gamma^{\text{out}}_{m,u,t},\ \forall (m,u) \in \mathcal{Z}, t, \tag{6c}$$

where $q_{m,u,t},\ q^{\text{in}}_{m,u,t},\ q^{\text{out}}_{m,u,t}$ denote the average flow rate, inflow and outflow rate of natural gas in the pipeline connecting nodes m and u, in absence of forecast errors and the non-negative variables $\gamma_{m,u,t},\ \gamma^{\text{in}}_{m,u,t},\ \gamma^{\text{out}}_{m,u,t}$ represent the changes in these flow rates during real-time in response to uncertainty.

The affine policies for the various flexible agents are shown in Fig. 1. In the following, we consider the more realistic case when forecasts are imperfect, i.e., $\mathbf{e}^\top \Omega_t \neq 0$. With the affine policies for the gas network operator in place, the equality constraints (2q)-(2t) governing the amount of linepack and evolution of linepack in gas pipelines hold true for any realization of Ω_t iff the following set of constraints holds true. For pipelines $\forall (m,u) \in \mathcal{Z}$,

$$h_{m,u,t} = H^0_{m,u} + q^{\text{in}}_{m,u,t} - q^{\text{out}}_{m,u,t},\ \ t = 1, \tag{7a}$$
$$h_{m,u,t} = h_{m,u,(t-1)} + (q^{\text{in}}_{m,u,t} - q^{\text{out}}_{m,u,t}),\ \ t > 1, \tag{7b}$$
$$q_{m,u,t} = \frac{q^{\text{in}}_{m,u,t} + q^{\text{out}}_{m,u,t}}{2};\ \gamma_{m,u,t} = \frac{\gamma^{\text{in}}_{m,u,t} + \gamma^{\text{out}}_{m,u,t}}{2},\ \ \forall t, \tag{7c}$$
$$\frac{S_{m,u}}{2}(\rho_{m,t} + \rho_{u,t} - \rho_{m,(t-1)} - \rho_{u,(t-1)}) = (\gamma^{\text{in}}_{m,u,t} - \gamma^{\text{out}}_{m,u,t}),\ \ t > 1, \tag{7d}$$

where $h_{m,u,t}$ is the nominal linepack in the pipeline in case perfect forecasts of wind power production were to be realized. Inter-temporal constraint (7d) governs the change in linepack as a response to uncertainty during real-time operation.

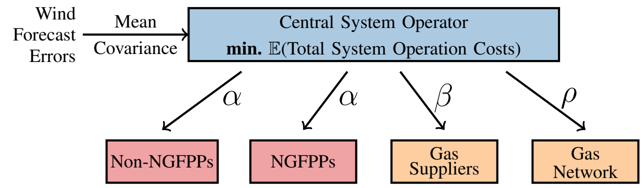

Fig. 1. Coordinated power and natural gas day-ahead dispatch

The nodal balance constraint for natural gas (2v) holds true for any realization of uncertainty Ω_t iff

$$\sum_{k \in \mathcal{A}^{\text{K}}_m} g_{k,t} - \sum_{i \in \mathcal{A}^{\text{G}}_m} \phi_i\ p_{i,t} - \sum_{u:(m,u) \in \mathcal{Z}} (q^{\text{in}}_{m,u,t} - q^{\text{out}}_{u,m,t}) = D^{\text{G}}_{m,t},\ \ \forall m,\ \forall t, \tag{8a}$$
$$\sum_{k \in \mathcal{A}^{\text{K}}_m} \beta_{k,t} - \sum_{i \in \mathcal{A}^{\text{G}}_m} \phi_i \alpha_{i,t} - \sum_{u:(m,u) \in \mathcal{Z}} (\gamma^{\text{in}}_{m,u,t} - \gamma^{\text{out}}_{u,m,t}) = 0\ \ \forall m,\ \forall t. \tag{8b}$$

Similarly, the power balance constraint in (2b) holds true for any realization of uncertainty Ω_t iff

$$\mathbf{e}^\top \mathbf{p}_t + \mathbf{e}^\top \mathbf{W}^{\text{PF}}_t = \mathbf{e}^\top \mathbf{D}^{\text{E}}_t,\ \ \forall t, \tag{9a}$$
$$\mathbf{e}^\top \boldsymbol{\alpha}_t = 1,\ \ \forall t. \tag{9b}$$

Constraints (8) and (9) are derived by separating the nominal and uncertainty-dependent terms in (2v) and (2b), respectively.

Given the affine policies in (5) and (6), constraints (2p) can be expanded as, $\forall (m,u) \in \mathcal{Z},\ \forall t$,

$$(q^2_{m,u,t} + (\mathbf{e}^\top \boldsymbol{\Omega}_t)^2 \gamma^2_{m,u,t} + 2(\mathbf{e}^\top \boldsymbol{\Omega}_t)\gamma_{m,u,t} q_{m,u,t}) = K^2_{m,u}(pr^2_{m,t} - pr^2_{u,t}) + (\mathbf{e}^\top \boldsymbol{\Omega}_t)^2 K^2_{m,u}(\rho^2_{m,t} - \rho^2_{u,t}) + 2(\mathbf{e}^\top \boldsymbol{\Omega}_t) K^2_{m,u}(\rho_{m,t} pr_{m,t} - \rho_{u,t} pr_{u,t}), \tag{10}$$

and then (10) can be replaced by the following set of constraints that must hold true for any realization of the uncertainty. For pipelines $\forall (m,u) \in \mathcal{Z},\ \forall t$,

$$q^2_{m,u,t} = K^2_{m,u}(pr^2_{m,t} - pr^2_{u,t}), \tag{11a}$$
$$\gamma^2_{m,u,t} = K^2_{m,u}(\rho^2_{m,t} - \rho^2_{u,t}), \tag{11b}$$
$$\gamma_{m,u,t} q_{m,u,t} = K^2_{m,u}(\rho_{m,t} pr_{m,t} - \rho_{u,t} pr_{u,t}). \tag{11c}$$

E. Power and Natural Gas Coordination with Affine Policies

In the following we present a finite-dimensional solvable approximation of the stochastic program (2), under the strategy of affine response to uncertainty. As shown in Fig. 1, this problem is solved by a central system operator during the day-ahead stage.

$$\min_{\Theta_2} \sum_{t \in \mathcal{T}} \Big(\sum_{i \in \mathcal{C}} C^{\text{E}}_i\ p_{i,t} + \sum_{k \in \mathcal{K}} C^{\text{G}}_k\ g_{k,t} \Big) \tag{12a}$$

subject to

$$\min_{\mathbb{P} \in \Pi} \mathbb{P}[p_{i,t} + (\mathbf{e}^\top \Omega_t)\alpha_{i,t} \geq \underline{P}_i] \geq (1 - \epsilon_i),\ \ \forall i,\ \forall t, \tag{12b}$$
$$\min_{\mathbb{P} \in \Pi} \mathbb{P}[p_{i,t} + (\mathbf{e}^\top \Omega_t)\alpha_{i,t} \leq \overline{P}_i] \geq (1 - \epsilon_i),\ \ \forall i,\ \forall t, \tag{12c}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[g_{k,t} + (\mathbf{e}^\top\Omega_t)\beta_{k,t} \geq \underline{G}_k] \geq (1-\epsilon_k),\ \forall k,\ \forall t, \tag{12d}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[g_{k,t} + (\mathbf{e}^\top\Omega_t)\beta_{k,t} \leq \overline{G}_k] \geq (1-\epsilon_k),\ \forall k,\ \forall t, \tag{12e}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[pr_{m,t} + (\mathbf{e}^\top\Omega_t)\rho_{m,t} \geq \underline{PR}_m] \geq (1-\epsilon_m), \forall m, \forall t, \tag{12f}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[pr_{m,t} + (\mathbf{e}^\top\Omega_t)\rho_{m,t} \leq \overline{PR}_m] \geq (1-\epsilon_m),\ \forall m,\ \forall t, \tag{12g}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[(pr_{u,t} - \Gamma_{m,u}\ pr_{m,t}) + (\mathbf{e}^\top\Omega_t)(\rho_{u,t} \tag{12h}$$

$$- \Gamma_{m,u}\ \rho_{m,t}) \leq 0] \geq (1-\epsilon_{mu}),\ \forall (m,u) \in \mathcal{Z}_c,\ \forall t, \tag{12i}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[q_{m,u,t} + (\mathbf{e}^\top\Omega_t)\gamma_{m,u,t} \geq 0] \geq (1-\epsilon_{mu}),\ \forall (m,u) \in \mathcal{Z},\ \forall t, \tag{12j}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[q^{\text{in}}_{m,u,t} + (\mathbf{e}^\top\Omega_t)\gamma^{\text{in}}_{m,u,t} \geq 0] \geq (1-\epsilon_{mu}),\ \forall (m,u) \in \mathcal{Z},\ \forall t, \tag{12k}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[q^{\text{out}}_{m,u,t} + (\mathbf{e}^\top\Omega_t)\gamma^{\text{out}}_{m,u,t} \geq 0] \geq (1-\epsilon_{mu}),\ \forall (m,u) \in \mathcal{Z},\ \forall t, \tag{12l}$$

$$\min_{\mathbb{P}\in\Pi} \mathbb{P}[h_{m,u,t} + S_{m,u}(\mathbf{e}^\top\Omega_t)\frac{\rho_{m,t}+\rho_{u,t}}{2} \geq H^0_{m,u}] \geq (1-\epsilon_{mu}),\ \forall (m,u) \in \mathcal{Z},\ t = |\mathcal{T}|, \tag{12m}$$

$$\text{(2c) - (2e)},\ (7),\ (8),\ (9),\ (11), \tag{12n}$$

where the optimization variables are $\Theta_2 = \{p_{i,t},\ \alpha_{i,t},\ g_{k,t},\ \beta_{k,t},\ pr_{m,t},\ \rho_{m,t},\ q_{m,u,t},\ \gamma_{m,u,t},\ q^{\text{in}}_{m,u,t},\ \gamma^{\text{in}}_{m,u,t},\ q^{\text{out}}_{m,u,t},\ \gamma^{\text{out}}_{m,u,t},\ h_{m,u,t}\}$. The expectation term in objective (2a) reduces to (12a) on account of the zero-mean assumption of Ω_t. As discussed in [16] for programs with a similar structure, the stochastic program given by (12) is computationally intractable due to the probabilistic distributionally robust chance constraints. To achieve tractability, a convex second-order cone (SOC) approximation of the non-convex individual distributionally robust chance constraints is adopted. Furthermore, the non-convex quadratic equality constraints (11) which represent the Weymouth equations for the uncertainty-aware gas flows require convexification. The approach towards solving (12) along with its final tractable form is discussed in the next section.

III. Solution Approach

A. SOC Reformulation of Probabilistic Constraints

For distributionally robust individual chance constraints under the assumption of known first and second-order moments of the underlying probability distribution, [17, Theorem 2.2] provides a SOC approximation based on a variant of Chebyshev's Inequality. While interested readers are directed to [17] for a proof, convex reformulation of constraint (12c) is presented below as an illustration.

With $\mathbf{\Sigma}^\Pi_t \in \mathbb{R}^{|\mathcal{J}|\times|\mathcal{J}|}$ as the t-th diagonal sub-matrix of the covariance matrix $\mathbf{\Sigma}^\Pi$ in time period t and $\mathbf{e} \in \mathbb{R}^{|\mathcal{J}|}$ denoting a vector of all ones, the probabilistic chance constraints (12c) can be approximated by the following SOC constraints

$$\sqrt{\frac{1-\epsilon_i}{\epsilon_i}}\left\|\alpha_{i,t}\mathbf{e}^\top(\mathbf{\Sigma}^\Pi_t)^{1/2}\right\|_2 \leq -p_{i,t} + \overline{P}_i,\ \forall i,\ \forall t. \tag{13}$$

Similar reformulation is performed for the other distributionally robust chance constraints in (12). References [18] and [19] remark that such conic reformulation based on Chebyshev's Inequality results in over-conservative solutions as $\epsilon_i \to 0$ while approaching infeasibility for $\epsilon_i \approx 0$. Exact reformulation of such chance constraints improving on this issue has been recently proposed in [18]. However, since the focus of this work is on uncertainty-aware coordination between electricity and natural gas systems, our formulation is limited to the conic approximation while ensuring that large enough risk measures $\epsilon_{(\cdot)}$ are considered in the case study (Section IV) such that infeasibility is avoided.

B. Convexification of Weymouth Equations

The non-convex quadratic equality constraints in (11a) can be equivalently written as

$$q^2_{m,u,t} \leq K^2_{m,u}(pr^2_{m,t} - pr^2_{u,t}),\ \forall (m,u) \in \mathcal{Z},\ \forall t, \tag{14a}$$

$$q^2_{m,u,t} \geq K^2_{m,u}(pr^2_{m,t} - pr^2_{u,t}),\ \forall (m,u) \in \mathcal{Z},\ \forall t. \tag{14b}$$

To relax (11a), we adopt the convex SOC constraints (14a) and drop the non-convex constraints (14b). The tightness of this relaxation is analysed in [20]. Note that (11b) can be convexified in the same manner. However, this convexification strategy cannot be applied to (11c) since the inequalities equivalent to (11c) are still non-convex. We adopt McCormick relaxation [21] defining rectangular envelopes around the bilinear terms in (11c) based on the known variable bounds. We first define auxiliary variables $\nu_{m,t}$ for gas nodes $m \in \mathcal{M}$ and $\lambda_{m,u,t}$ for the pipelines $(m,u) \in \mathcal{Z},\ \forall t$ and then replace (11c) by the following set of constraints:

$$\lambda_{m,u,t} - K^2_{m,u}\nu_{m,t} + K^2_{m,u}\nu_{u,t} = 0,\ \forall (m,u) \in \mathcal{Z},\ \forall t, \tag{15a}$$

$$\lambda_{m,u,t} = q_{m,u,t}\gamma_{m,u,t}, \forall (m,u) \in \mathcal{Z}, \forall t, \tag{15b}$$

$$\nu_{m,t} = pr_{m,t}\rho_{m,t},\ \forall m,\ \forall t, \tag{15c}$$

$$\nu_{u,t} = pr_{u,t}\rho_{u,t},\ \forall u : (m,u) \in \mathcal{Z},\ \forall t. \tag{15d}$$

To illustrate the McCormick relaxation, the inequalities that form rectangular bounds around and replace the non-convex constraints (15c) are

$$\forall m,t \begin{cases} \rho^L_{m,t}pr_{m,t} + pr^L_{m,t}\rho_{m,t} \leq \nu_{m,t} + pr^L_{m,t}\rho^L_{m,t} \\ \rho^U_{m,t}pr_{m,t} + pr^U_{m,t}\rho_{m,t} \leq \nu_{m,t} + pr^U_{m,t}\rho^U_{m,t} \\ \rho^L_{m,t}pr_{m,t} + pr^U_{m,t}\rho_{m,t} \leq \nu_{m,t} + pr^U_{m,t}\rho^L_{m,t} \\ \rho^U_{m,t}pr_{m,t} + pr^L_{m,t}\rho_{m,t} \leq \nu_{m,t} + pr^L_{m,t}\rho^U_{m,t} \end{cases} \tag{16}$$

where the superscripts L and U indicate lower and upper bounds of the variables, respectively.

Following the convex approximation of probabilistic constraints and relaxation of Weymouth equations, the tractable form of the distributionally robust chance-constrained day-ahead coordinated power and natural gas dispatch is presented in Appendix A. Problem (18) is a convex second-order cone program (SOCP) and is solvable using off-the-shelf convex optimization solvers.

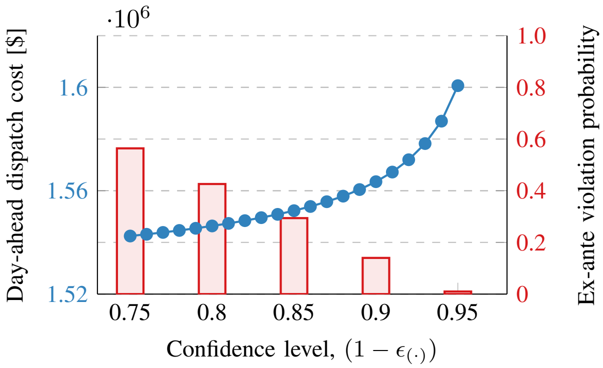

Fig. 2. Day-ahead dispatch cost with values of $\epsilon_{(\cdot)}$ chosen for the distributionally robust chance constraints is shown by line with markers ●. The ex-ante violation probability of these constraints, evaluated for 1,000 test samples, is shown in bars.

IV. Case Study

A coupled power and natural gas system consisting of a 12-node gas network connected to the IEEE 24-bus reliability test system [7] is used to evaluate our proposed coordinated dispatch. The installed wind capacity reaches two-thirds of the peak demand in the simulation horizon of 24 hours. A dataset of 1,000 zero-mean wind forecast error scenarios based on actual measurements recorded in Western Denmark [22] is used to empirically estimate the covariance matrix $\mathbf{\Sigma}^{\Pi}$. Data for the parameters of the power and natural gas networks and for the operational characteristics of all assets in the systems are provided in online appendix [23]. The parameters $\epsilon_{(\cdot)}$ for all distributionally robust constraints in (18) are set to identical values. The problem is implemented in Julia v1.1.1 modeled with JuMP v0.2 and solved to optimality by Mosek v9.0 with an average CPU time of 1.67 seconds on a personal computer with 8GB memory running on Intel Core i5 clocked at 2.3 GHz. The optimal solution provides nominal dispatch schedule as well as affine policies that quantify the response to uncertain wind realizations during real-time.

In order to evaluate the quality of the solution obtained and to make an informed choice for $\epsilon_{(\cdot)}$, we perform *ex-ante* simulations using a test dataset of wind realization scenarios, distinct from those used to estimate the covariance matrix. With the day-ahead decisions, i.e., nominal production schedules and affine policies fixed to their optimal values obtained, we compute the violation probability of the distributionally robust chance constraints (12b)-(12m) and (2d)-(2e) for a choice of $\epsilon_{(\cdot)}$ as

$$\eta_{\epsilon} = \frac{1}{N_s}\sum_{s=1}^{N_s} \mathbb{I}_s, \qquad (17)$$

where $\mathbb{I}_s$ is the indicator function that takes a value 1 if at least one of these constraints is violated for the wind realization that corresponds to scenario s. The lineplot in Fig. 2 shows the expected cost of day-ahead dispatch at various values of confidence levels $(1 - \epsilon_{(\cdot)})$ imposed on the probabilistic constraints. With a higher confidence of meeting the constraints, the expected cost of day-ahead dispatch increases. The bars show the ex-ante violation probability computed at selected confidence level values. For $\epsilon_{(\cdot)} = 0.05$, an ex-ante violation probability of 0.01 is expected at a day-ahead expected dispatch cost of $1,600,000. Fig. 3 shows the optimal affine policies obtained for $\epsilon_{(\cdot)} = 0.05$.

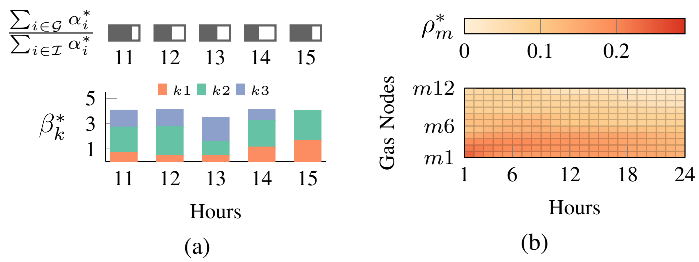

Fig. 3. Optimal affine policies for $\epsilon_{(\cdot)} = 0.05$. While (a) shows shares of affine policies α^* allocated to NGFPPs and policies β^* allocated to natural gas suppliers, (b) shows values of ρ^* at the 12 gas nodes in the system.

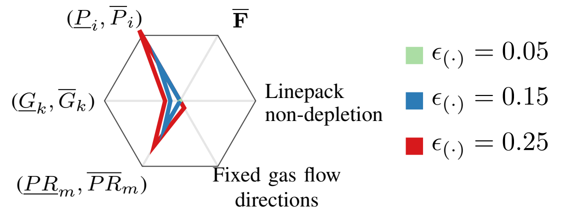

Fig. 4. Ex-ante violation probabilities by constraint type. Corners of the hexagon represent a probability of 0.5.

To develop a better understanding of the constraint violations, we use the same metric to compute violation probabilities for each of the following sets of constraints: generator bounds (12b) and (12c), line flow limits (2d) and (2e), gas supplier bounds (12d) and (12e), nodal gas pressure bounds (12f) and (12g), natural gas flow direction constraints (12j)-(12l), and non-depletion of linepack in pipelines requirement (12m). Fig. 4 shows the probability of violation of these individual constraints for different choices of $\epsilon_{(\cdot)}$. It can be observed that the power generation limits (12b) and (12c) and the nodal gas pressure limits (12f) and (12g) are most susceptible to violation. Only at the highest considered value of $\epsilon_{(\cdot)} = 0.25$, we find a small violation probability of power flow limits (0.004) in the transmission lines and of the pre-determined gas flow directions (0.054). This shows that the power lines are not congested and the gas flow directions assumed were correctly anticipated. Furthermore, the non-depletion of linepack constraints are satisfied even at $\epsilon_{(\cdot)} = 0.25$. This indicates that there is enough short-term gas storage available in the gas pipelines such that they are not depleted at the end of the day while providing flexibility to the power systems. However, it should be noted that these outcomes are system specific.

V. Conclusion

We proposed a distributionally robust chance-constrained coordination between power and natural gas systems to study the propagation of uncertainty from the power to the gas side. Our tractable reformulation of the stochastic program including recourse actions from the natural gas system in the form of affine policies results in a convex SOCP. Ex-ante out-of-sample evaluations are used to demonstrate the quality of the solution while highlighting a trade-off between dispatch cost and violation probability, which influences the choice of $\epsilon_{(\cdot)}$. The coordination model enables efficient harnessing of short-term flexibility from the assets in natural gas networks for the power systems facing uncertainty.

Analyzing the proposed coordination in a market context wherein payments for the provision of flexibility-as-a-service are considered, is an interesting topic to investigate in future. Further, more detailed out-of-sample simulations should be performed to understand the quality of optimal affine responses and their impact on the feasibility of the power and natural gas network physical constraints.

Appendix A

$$\min_{\Theta_2 \cup \{\lambda_{m,u,t},\ \nu_{m,t}\}} \sum_{t\in\mathcal{T}} \Big(\sum_{i\in\mathcal{C}} C_i^{\mathrm{E}}\ p_{i,t} + \sum_{k\in\mathcal{K}} C_k^{\mathrm{G}}\ g_{k,t} \Big) \tag{18a}$$

subject to

$$\xi_i \Big\| -\alpha_{i,t}\mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \Big\|_2 \leq p_{i,t} - \underline{P}_i,\ \forall i,\ \forall t, \tag{18b}$$

$$\xi_i \Big\| \alpha_{i,t}\mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \Big\|_2 \leq -p_{i,t} + \overline{P}_i,\ \forall i,\ \forall t, \tag{18c}$$

$$\xi_{nr} \Big\| \{\mathbf{\Psi}(\mathbf{\Psi}_{\mathrm{I}}\boldsymbol{\alpha}_t\mathbf{e}^\top - \mathbf{\Psi}_{\mathrm{J}})\}_{(n,r)} (\mathbf{\Sigma}_t^{\Pi})^{1/2} \Big\|_2 \leq \{\overline{\mathbf{F}} + \mathbf{\Psi}(\mathbf{\Psi}_{\mathrm{D}}\mathbf{D}_t^{\mathrm{E}} - \mathbf{\Psi}_{\mathrm{I}}\mathbf{p}_t - \mathbf{\Psi}_{\mathrm{J}}\mathbf{W}_t^{\mathrm{PF}})\}_{(n,r)},\ \forall (n,r)\in\mathcal{L},\ \forall t, \tag{18d}$$

$$\xi_{nr} \Big\| -\{\mathbf{\Psi}(\mathbf{\Psi}_{\mathrm{I}}\boldsymbol{\alpha}_t\mathbf{e}^\top - \mathbf{\Psi}_{\mathrm{J}})\}_{(n,r)} (\mathbf{\Sigma}_t^{\Pi})^{1/2} \Big\|_2 \leq \{\overline{\mathbf{F}} - \mathbf{\Psi}(\mathbf{\Psi}_{\mathrm{D}}\mathbf{D}_t^{\mathrm{E}} - \mathbf{\Psi}_{\mathrm{I}}\mathbf{p}_t - \mathbf{\Psi}_{\mathrm{J}}\mathbf{W}_t^{\mathrm{PF}})\}_{(n,r)},\ \forall (n,r)\in\mathcal{L},\ \forall t, \tag{18e}$$

$$\xi_k \Big\| -\beta_{k,t}\mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \Big\|_2 \leq g_{k,t} - \underline{G}_i,\ \forall k,\ \forall t, \tag{18f}$$

$$\xi_k \Big\| \beta_{k,t}\mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \Big\|_2 \leq -g_{k,t} + \overline{G}_i,\ \forall k,\ \forall t, \tag{18g}$$

$$\xi_m \Big\| -\rho_{m,t}\mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \Big\|_2 \leq pr_{m,t} - \underline{PR}_m,\ \forall m,\ \forall t, \tag{18h}$$

$$\xi_m \Big\| \rho_{m,t}\mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \Big\|_2 \leq -pr_{m,t} + \overline{PR}_m,\ \forall m,\ \forall t, \tag{18i}$$

$$\xi_{mu} \Big\| (\rho_{u,t} - \Gamma_{m,u}\rho_{m,t})\mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \Big\|_2 \leq \Gamma_{m,u}pr_{m,t} - pr_{u,t},\ \forall (m,u)\in\mathcal{Z}_c,\ \forall t, \tag{18j}$$

$$\xi_{mu} \Big\| -\gamma_{m,u,t}\mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \Big\|_2 \leq q_{m,u,t}, \forall (m,u)\in\mathcal{Z}, \forall t, \tag{18k}$$

$$\xi_{mu} \Big\| -\gamma_{m,u,t}^{\mathrm{in}}\mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \Big\|_2 \leq q_{m,u,t}^{\mathrm{in}}, \forall (m,u)\in\mathcal{Z}, \forall t, \tag{18l}$$

$$\xi_{mu} \Big\| -\gamma_{m,u,t}^{\mathrm{out}}\mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \Big\|_2 \leq q_{m,u,t}^{\mathrm{out}}, \forall (m,u)\in\mathcal{Z}, \forall t, \tag{18m}$$

$$\gamma_{m,u,t}^2 \leq K_{m,u}^2(\rho_{m,t}^2 - \rho_{u,t}^2),\ \forall (m,u)\in\mathcal{Z},\ \forall t, \tag{18n}$$

$$\xi_{mu} \Big\| -(\rho_{m,t} + \rho_{u,t})(\frac{S_{m,u}}{2})^{\frac{1}{2}}\mathbf{e}^\top (\mathbf{\Sigma}_t^{\Pi})^{1/2} \Big\|_2 \leq h_{m,u,t} - H_{m,u}^0,\ \forall (m,u)\in\mathcal{Z},\ t = |\mathcal{T}|, \tag{18o}$$

McCormick envelopes of (15b) and (15d), (18p)

(7), (8), (9), (14a), (15a), (16), (18q)

where $\xi_i = \sqrt{\frac{1-\epsilon_i}{\epsilon_i}},\ \xi_{nr} = \sqrt{\frac{1-\epsilon_{nr}}{\epsilon_{nr}}},\ \xi_k = \sqrt{\frac{1-\epsilon_k}{\epsilon_k}},\ \xi_m = \sqrt{\frac{1-\epsilon_m}{\epsilon_m}},\ \xi_{mu} = \sqrt{\frac{1-\epsilon_{mu}}{\epsilon_{mu}}}$ are parameters.

References

[1] S. Chen, A. J. Conejo, R. Sioshansi, and Z. Wei, "Unit commitment with an enhanced natural gas-flow model," *IEEE Transactions on Power Systems*, 2019, to be published.

[2] P. Pinson, L. Mitridati, C. Ordoudis, and J. Østergaard, "Towards fully renewable energy systems: Experience and trends in Denmark," *CSEE Journal of Power and Energy Systems*, vol. 3, no. 1, pp. 26–35, 2017.

[3] G. Byeon and P. Van Hentenryck, "Unit commitment with gas network awareness," *IEEE Transactions on Power Systems*, 2019, to be published.

[4] A. Zlotnik, L. Roald, S. Backhaus, M. Chertkov, and G. Andersson, "Coordinated scheduling for interdependent electric power and natural gas infrastructures," *IEEE Transactions on Power Systems*, vol. 32, no. 1, pp. 600–610, 2017.

[5] S. Chen, A. J. Conejo, R. Sioshansi, and Z. Wei, "Operational equilibria of electric and natural gas systems with limited information interchange," *IEEE Transactions on Power Systems*, 2019, to be published.

[6] C. M. Correa-Posada and P. Sánchez-Martín, "Integrated power and natural gas model for energy adequacy in short-term operation," *IEEE Transactions on Power Systems*, vol. 30, no. 6, pp. 3347–3355, 2015.

[7] C. Ordoudis, P. Pinson, and J. M. Morales, "An integrated market for electricity and natural gas systems with stochastic power producers," *European Journal of Operational Research*, vol. 272, no. 2, pp. 642–654, 2019.

[8] H. Ameli, M. Qadrdan, and G. Strbac, "Coordinated operation strategies for natural gas and power systems in presence of gas-related flexibilities," *Energy Systems Integration*, vol. 1, no. 1, pp. 3–13, 2019.

[9] A. Schwele, C. Ordoudis, J. Kazempour, and P. Pinson, "Coordination of power and natural gas systems: Convexification approaches for linepack modeling," in *Proc. 13th IEEE PES PowerTech Conference*, Milan, 2019.

[10] C. Wang, W. Wei, J. Wang, and T. Bi, "Convex optimization based adjustable robust dispatch for integrated electric-gas systems considering gas delivery priority," *Applied Energy*, vol. 239, pp. 70–82, 2019.

[11] B. Odetayo, J. MacCormack, W. Rosehart, and H. Zareipour, "A chance constrained programming approach to integrated planning of distributed power generation and natural gas network," *Electric Power Systems Research*, vol. 151, pp. 197–207, 2017.

[12] A. Ratha, J. Kazempour, A. Virag, and P. Pinson, "Exploring market properties of policy-based reserve procurement for power systems," in *Proc. 58th IEEE Conference on Decision and Control*, Nice, 2019.

[13] R. D. Christie, B. F. Wollenberg, and I. Wangensteen, "Transmission management in the deregulated environment," *Proceedings of the IEEE*, vol. 88, no. 2, pp. 170–195, 2000.

[14] A. Georghiou, D. Kuhn, and W. Wiesemann, "The decision rule approach to optimization under uncertainty: Methodology and applications," *Computational Management Science*, 2018.

[15] A. Georghiou, W. Wiesemann, and D. Kuhn, "Generalized decision rule approximations for stochastic programming via liftings," *Mathematical Programming*, vol. 152, no. 1, pp. 301–338, 2015.

[16] S. Zymler, D. Kuhn, and B. Rustem, "Distributionally robust joint chance constraints with second-order moment information," *Mathematical Programming*, vol. 137, no. 1, pp. 167–198, 2013.

[17] M. R. Wagner, "Stochastic 0-1 linear programming under limited distributional information," *Operations Research Letters*, vol. 36, no. 2, pp. 150 – 156, 2008.

[18] W. Xie and S. Ahmed, "Distributionally robust chance constrained optimal power flow with renewables: A conic reformulation," *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 1860–1867, 2018.

[19] Y. Dvorkin, "A chance-constrained stochastic electricity market," *arXiv e-prints*, p. arXiv:1906.06963, 2019.

[20] C. Borraz-Snchez, R. Bent, S. Backhaus, H. Hijazi, and P. Van Hentenryck, "Convex relaxations for gas expansion planning," *INFORMS Journal on Computing*, vol. 28, no. 4, pp. 645–656, 2016.

[21] G. P. McCormick, "Computability of global solutions to factorable nonconvex programs: Part I - Convex underestimating problems," *Mathematical Programming*, vol. 10, no. 1, pp. 147–175, 1976.

[22] P. Pinson, "Wind energy: Forecasting challenges for its operational management," *Statistical Science*, vol. 28, no. 4, pp. 564–585, 11 2013.

[23] A. Ratha, A. Schwele, J. Kazempour, P. Pinson, S. S. Torbaghan, and A. Virag, "Online appendix for paper: Data-driven affine policies for flexibility provision by natural gas networks to power systems," 2019. [Online]. Available: https://doi.org/10.5281/zenodo.3465499

***Conflict of Interest Form**

Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: